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Domination and Chromatic Number of Pan Graph and Lollipop Graph

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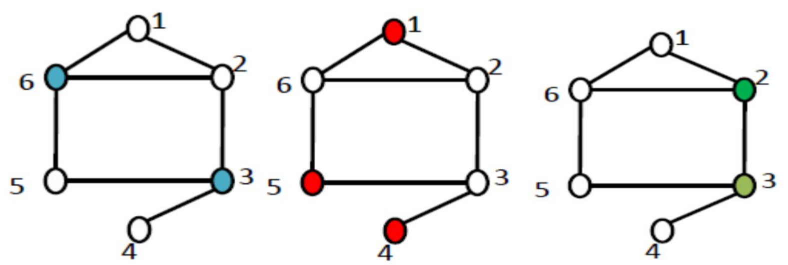
Abstract—***Domination number and Chromatic number are important characteristics of a graph. Lollipop and Pan Graphs are special types of Graphs. A dominating set of a graph*** $G=(V,E)$ ***is a subset*** D ***of*** E ***such that every vertex not in*** D ***is adjacent to atleast one vertex of*** D***. The domination number*** $\gamma(G)$ ***of a graph*** G ***is the cardinality of a smallest dominating set.In this paper, we determine the domination number and chromatic number of pan graph and lollipop graph.***
Keywords— ***Domination number, Chromatic number, Pan graph, Lollipop graph.***

I. INTRODUCTION

Domination Number

Let $\boldsymbol{G=(V,E)}$ be a graph, a subset $\boldsymbol{D}$ of $\boldsymbol{V(G)}$ is said to dominating set for a graph $\boldsymbol{G}$ if every vertex not in $\boldsymbol{D}$ is adjacent to at least one member of $\boldsymbol{D}$. The domination number $\boldsymbol{\gamma(G)}$ is the number of vertices in a smallest dominating set for $\boldsymbol{G}$.



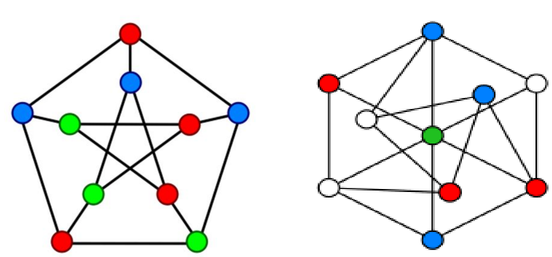
(a) (b) (c)

Fig 1.1 Domination Number

In this example $\boldsymbol{V(G)=\{1,2,3,4,5,6\}}$ Subset $\boldsymbol{D}$ for Graph (a), (b) and (c) are $\boldsymbol{\{6,3\},\{1,5,6\}}$ and $\boldsymbol{\{2,3\}}$. So its vertex dominating number $\boldsymbol{\gamma(G)=2.}$

Chromatic Number

The Chromatic number of a graph is the minimum number of colours needed to colour the vertices of Graph so that no two adjacent vertices share the same colour. It is denoted by $\chi(G)$.



$\chi(G)=3$ $\chi(G)=4$

Fig.1.2 Chromatic Number

Pan Graph

The pan graph is the graphobtained by joining a cycle graph C_n to a singleton graph K_1 with a bridge. It is denoted by P_n.

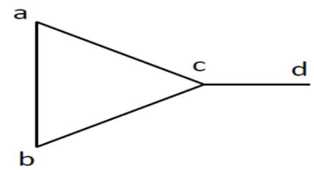
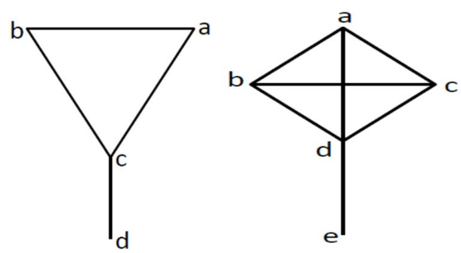


Fig. 1.3 Pan graph

Lollipop Graph

The lollipop graph is the graph obtained by joining a complete graph K_m to a path graph P_n with a bridge. It is denoted by $L_{m,n}$.



$L_{3,1}$ $L_{4,1}$

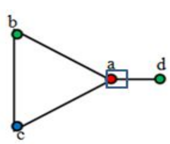
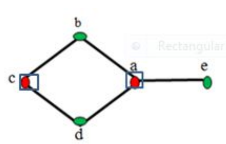
Fig. 1.4 Lollipop Graph

II. RESULT AND DISCUSSION

We used induction method to obtained Domination number and Chromatic number for Pan Graph

PAN GRAPH

TABLE 2.1 PAN GRAPH

S. No.	Pan Graph (P_n)	Graph	$\gamma(G)$	$\chi(G)$
1	$n=3, P_3$		$\gamma(P_3)=1$	$\chi(P_3)=3$
2	$n=4, P_4$		$\gamma(P_4)=2$	$\chi(P_4)=2$

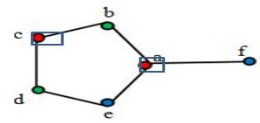
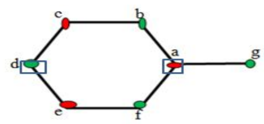
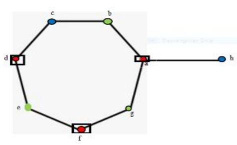
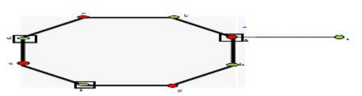
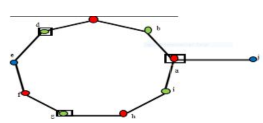
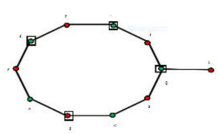
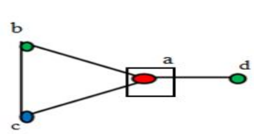
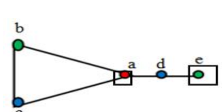
3	$n=5, P_5$		$\gamma(P_5)=2$	$\chi(P_5)=3$
4	$n=6, P_6$		$\gamma(P_6)=2$	$\gamma(P_6)=2$
5	$n=7, P_7$		$\gamma(P_7)=3$	$\gamma(P_7)=3$
6	$n=8, P_8$		$\gamma(P_8)=3$	$\gamma(P_8)=2$
7	$n=9, P_9$		$\gamma(P_9)=3$	$\gamma(P_9)=3$
8	$n=10, P_{10}$		$\gamma(P_{10})=4$	$\gamma(P_{10})=2$

TABLE 2.2 LOLLIPOP GRAPH ($L_{3,n}$)

S. No.	Lollipop ($L_{3,n}$)	Graph	$\gamma(G)$	$\chi(G)$
1	$L_{3,1}$		$\gamma(L_{3,1})=1$	$\chi(L_{3,1})=3$
2	$L_{3,2}$		$\gamma(L_{3,2})=2$	$\chi(L_{3,2})=3$

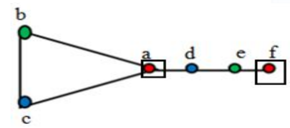
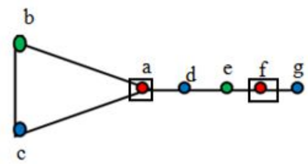
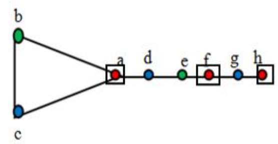
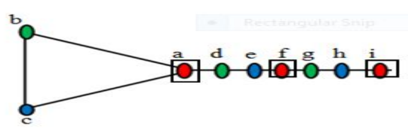
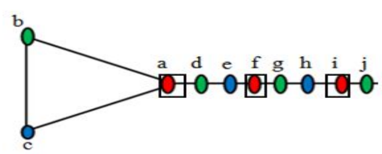
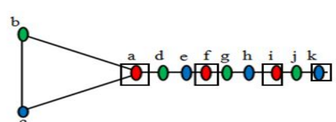
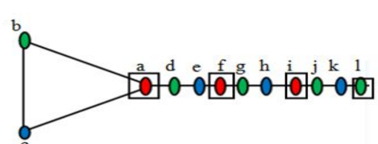
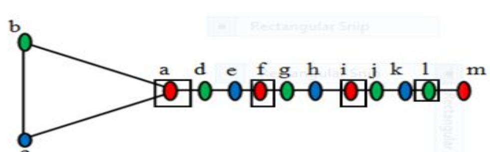
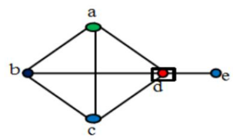
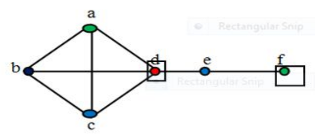
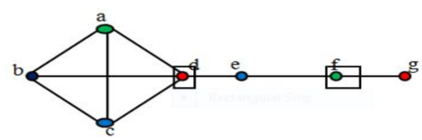
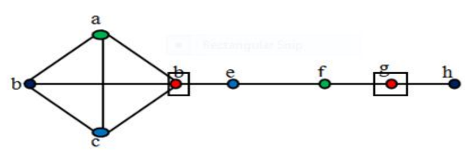
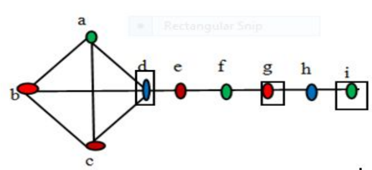
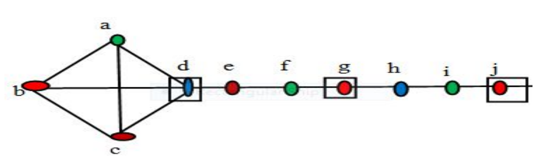
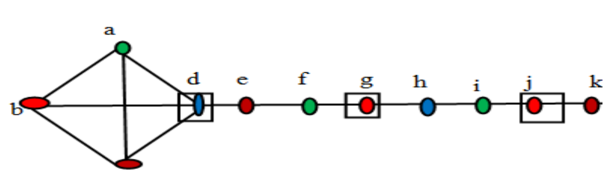
3	$L_{3,3}$		$\gamma(L_{3,3}) = 2$	$\chi(L_{3,3}) = 3$
4	$L_{3,4}$		$\gamma(L_{3,4}) = 2$	$\chi(L_{3,4}) = 3$
5	$L_{3,5}$		$\gamma(L_{3,5}) = 3$	$\chi(L_{3,5}) = 3$
6	$L_{3,6}$		$\gamma(L_{3,6}) = 3$	$\chi(L_{3,6}) = 3$
7	$L_{3,7}$		$\gamma(L_{3,7}) = 3$	$\chi(L_{3,7}) = 3$
8	$L_{3,8}$		$\gamma(L_{3,8}) = 4$	$\chi(L_{3,8}) = 3$
9	$L_{3,9}$		$\gamma(L_{3,9}) = 4$	$\chi(L_{3,9}) = 3$
10	$L_{3,10}$		$\gamma(L_{3,10}) = 4$	$\chi(L_{3,10}) = 3$

TABLE 2.3 LOLLIPOP GRAPH ($L_{4,n}$)

S. No.	Lollipop ($L_{4,n}$)	Graph	$\gamma(G)$	$\chi(G)$
1	$L_{4,1}$		$\gamma(L_{4,1})=1$	$\chi(L_{4,1})=4$
2	$L_{4,2}$		$\gamma(L_{4,2})=2$	$\chi(L_{4,2})=4$
3	$L_{4,3}$		$\gamma(L_{4,3})=2$	$\chi(L_{4,3})=4$
4	$L_{4,4}$		$\gamma(L_{4,4})=2$	$\chi(L_{4,4})=4$
5	$L_{4,5}$		$\gamma(L_{4,5})=3$	$\chi(L_{4,5})=4$
6	$L_{4,6}$		$\gamma(L_{4,6})=3$	$\chi(L_{4,6})=4$
7	$L_{4,7}$		$\gamma(L_{4,7})=3$	$\chi(L_{4,7})=4$

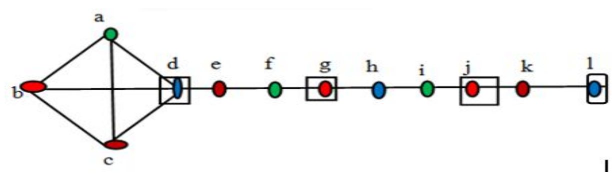
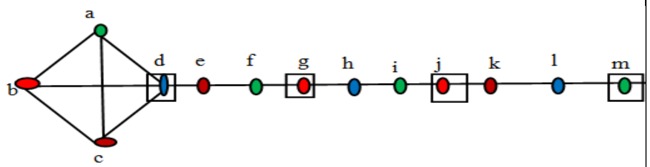
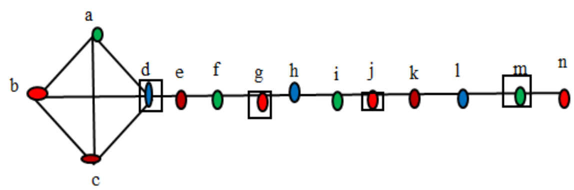
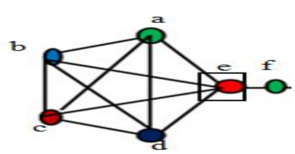
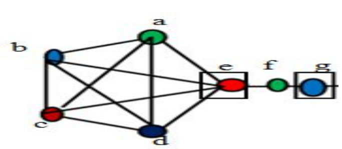
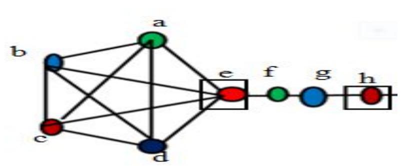
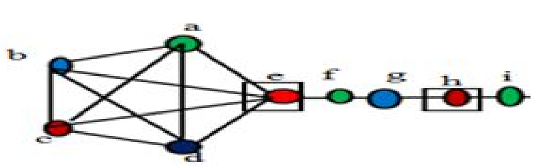
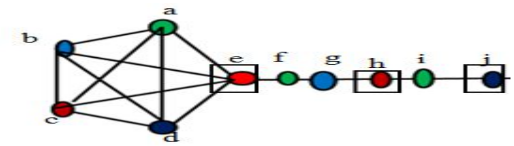
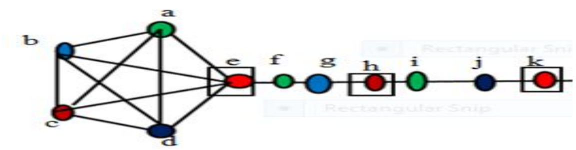
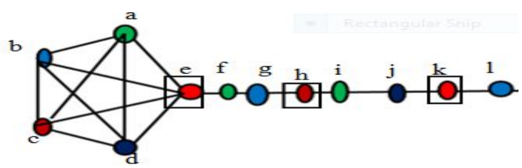
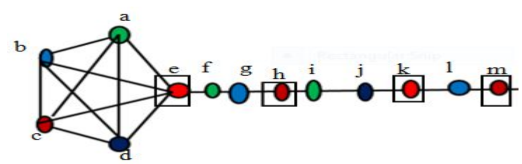
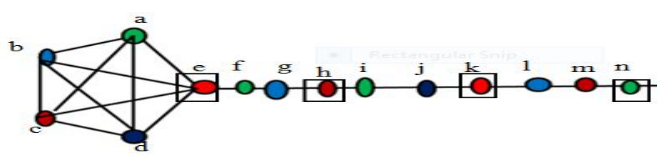
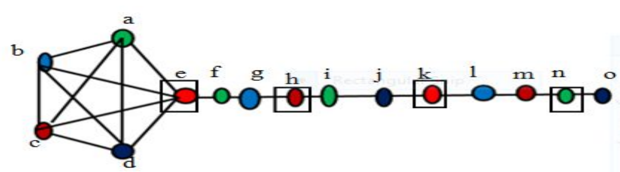
8	$L_{4,8}$		$\gamma(L_{4,8})=4$	$\chi(L_{4,8})=4$
9	$L_{4,9}$		$\gamma(L_{4,9})=4$	$\chi(L_{4,9})=4$
10	$L_{4,10}$		$\gamma(L_{4,10})=4$	$\chi(L_{4,10})=4$

TABLE 2.4 LOLLIPOP GRAPH ($L_{5,n}$)

S. No.	Lollipop ($L_{5,n}$)	Graph	$\gamma(G)$	$\chi(G)$
1	$L_{5,1}$		$\gamma(L_{5,1})=1$	$\chi(L_{5,1})=5$
2	$L_{5,2}$		$\gamma(L_{5,2})=2$	$\chi(L_{5,2})=5$
3	$L_{5,3}$		$\gamma(L_{5,3})=2$	$\chi(L_{5,3})=5$
4	$L_{5,4}$		$\gamma(L_{5,4})=2$	$\chi(L_{5,4})=5$

5	$L_{5,5}$		$\gamma(L_{5,5})=3$	$\chi(L_{5,5})=5$
6	$L_{5,6}$		$\gamma(L_{5,6})=3$	$\chi(L_{5,6})=5$
7	$L_{5,7}$		$\gamma(L_{5,7})=3$	$\chi(L_{5,7})=5$
8	$L_{5,8}$		$\gamma(L_{5,8})=4$	$\chi(L_{5,8})=5$
9	$L_{5,9}$		$\gamma(L_{5,9})=4$	$\chi(L_{5,9})=5$
10	$L_{5,10}$		$\gamma(L_{5,10})=4$	$\chi(L_{5,10})=5$

III. CONCLUSION

From the Table 2.1, we conclude that

a) Chromatic number for Pan Graph P_n is

$$\chi(P_n)=3, if\ \ n\ is\ odd$$
$$=2, if\ \ n\ is\ \ even$$

b) Domination number for Pan Graph P_n is

$$\Upsilon(P_n)=\frac{n}{3}\ \ for\ 3n\ \ i.e., 3,6,9,12,\ldots$$
$$=\left\lceil\frac{n}{3}\right\rceil\ \ \ for\ others$$

Where,⌈ ⌉ is defined as the greatest integer.

From the Table 2.2, 2.3 and 2.4 we conclude that

a) Chromatic number for Lollipop graph $L_{m,n}$ is $\chi(L_{m,n}) = m$

b) Domination number for Lollipop Graph $L_{m,n}$ is $\gamma(L_{m,n}) = \left\lceil \frac{n+2}{3} \right\rceil$

Where, is ⌈ ⌉ defined as the greatest integer.

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